The Outlook for Simulation of Forward Flight Effects on Aircraft Noise

D. G. Crighton*
University of Leeds, England
J. E. Ffowcs Williams†
University of Cambridge, England
I. C. Cheeseman‡
University of Southampton, England

This paper offers a wide-ranging examination of the fundamental issues behind recent efforts to devise means by which forward flight effects on aircraft noise may be simulated in ground-based facilities. Theoretical predictions of flight effects for simple configurations are noted, the advantages and disadvantages of various types of simulation facility are set out, and the features of possible noise sources and propagation mechanisms are tabulated. Opinions are then given as to how these sources and mechanisms may best be simulated, and the paper concludes with both general and very specific recommendations for future experimental and theoretical work.

I. Introduction and General Background

EFFORTS are currently being made to adapt existing facilities, and to design new ones, for the study of forward flight effects on aircraft noise. The flight speed range of interest is limited to that relevant to aircraft landing and takeoff operations, and thus involves flight Mach numbers up to, say 0.3. In many cases the flight is simulated with a static noise source surrounded by a limited volume of moving fluid, and in almost all cases the simulation involves a significant reduction of physical scale. There are clearly considerable difficulties in the way of such flight simulations for acoustic purposes; indeed, as this paper shows, it is clear that some aspects of the noise field of a full-scale aircraft in flight can probably never be simulated in any way essentially different from actual flight at full scale, regardless of any limitations (or otherwise) that may apply to particular facilities. Existing studies of forward flight facilities have hardly recognized these essential difficulties of principle, and have concerned themselves with more obvious practical prerequisites such as low self-noise levels from wind-tunnel fans and tunnel wall linings. Here, a more fundamental assessment is attempted. While a discussion is included of the limitations of particular types of facility, the paper concentrates far more on breaking the noise field of an aircraft into more or less independent mechanisms and considering for each of these the following questions: 1) Has the mechanism been positively identified under static conditions and under flight conditions? 2) Can the flight level be predicted to within 3 dB on the basis of an accurate static measurement? and 3) Has the mechanism a known counterpart at smaller length scales? Underlying the discussion is a recognition of the implications of recent theoretical studies of flight effects on the sound field radiated by very simple real sources. The results of these studies are reported briefly here in the Appendix; they show, among many things, that flight effects depend delicately on the detailed nature of a source, that flight effects are more varied and are generally much larger than would have been anticipated on the basis of previous ideas, and that acoustic effects due to flight convection do not by any means always vanish at 90 deg to the flight path, in contradiction to a very widely accepted, though undoubtedly false, idea.

Various kinds of facilities have been examined in the past few years in an attempt to simulate the noise field associated with an aircraft flyover. These include wind tunnels, whirling rotor arms, tracked vehicles, trucks, and taxiing aircraft. In most of these simulations difficulties of two kinds arise. First, there are the difficulties of practical technique, some of which have to be faced in any case in aircraft flyover noise tests at full scale, others of which are associated with the particular mode of simulation (such as, for example, the measurement of unsteady pressure in a moving wind-tunnel stream and the difficulty of accounting for path curvature on the Rolls-Royce Spinning Rig at Aston Down). Second, there are difficulties of principle concerning the corrections which must be applied to the results from the simulation in order to obtain the results for the supposedly equivalent flyover. For example, if pressure measurements are taken at a point fixed relative to the model within a wind tunnel, there are the usual Doppler factors to be used in relating frequency, distance, and angle in the simulation to frequency, distance, and angle in the flyover. If the pressure measurements are taken outside an open-jet tunnel there is, in addition, the issue of sound transmission across the open-jet shear layer to be faced. In the facilities which give the more straightforward simulation using ground-running vehicles or aircraft, there are corrections to be applied for ground reflection effects, which are usually much more acute than in the flyover. In almost all cases there is also the problem of deciding whether a smallscale simulation facility preserves the necessary dimensionless parameters of the full-scale flyover.

This second class of difficulties really represents the greater obstacle, for the corrections at stake cannot, by definition, be found accurately from any experiment, while theoretical estimates of the corrections can only be based on simplified model calculations. The most suitable type of facility is therefore one in which the corrections required to reach the equivalent flyover situation are smallest or most easily estimated correctly and unambiguously from simple arguments. It would be prudent to require that the corrections for converting the results from a flight simulation test to

Presented as Paper 76-530 at the 3rd AIAA Aero-Acoustics Conference, Palo Alto, Calif., July 20-23, 1976; submitted Oct. 12, 1976; revision received May 31, 1977.

Index categories: Aeroacoustics; Noise.

^{*}Professor of Applied Mathematics, Head of Department of Applied Mathematical Studies.

[†]Rank Professor of Engineering (Acoustics), University Engineering Department. Associate Fellow AIAA.

[‡]Westland Professor of Helicopter Engineering, Department of Aeronautics and Astronautics.

actual flight should not exceed 3 dB, the order of magnitude of what we consider the tolerable error band in a practical case.

As an example of the magnitude of the corrections which may have to be applied to convert simulation into flight, the work carried out in the NASA Ames 40- ×80-ft tunnel using several different full-scale research aircraft may be cited. The flight noise levels were measured directly and compared with those measured in the reverberant tunnel, and all obvious corrections were applied to both sets of data (e.g., Doppler corrections to the tunnel frequencies, ground reflexion and atmospheric attenuation corrections to the flight levels, etc.). These corrections were only of the order of 3-5 dB and left a discrepancy of some 12-15 dB across the whole of the audible spectrum. This, however, could largely be accounted for by calibrating the tunnel in the presence of flow and determining the tunnel effect on the acoustic field of a source whose freefield was known in detail. A "reverberation" correction for each 1/3-octave band was deduced in this way for each value of the flight Mach number, and the tunnel levels, after correction for reverberation, were then found to agree rather well with the flight levels. There are obvious dangers in this procedure, however, as evidenced by the fact that in particular cases the flight and tunnel levels still differed by as much as 7 dB.

This report might well seem rather destructive in that it uncovers all sorts of difficulties in the way of flight simulation. Some of our points may, in fact, turn out to be entirely academic, but we consider it unlikely that there can be any rational grounds for avoiding the following contentions: 1) that an all-purpose flight simulation facility is permanently quite out of the question; 2) that simulation will become very much easier and more reliable when certain fundamental theoretical and experimental work has been carried out on the scaling laws for sources and propagation mechanisms under static and flight conditions; and 3) that objections raised here as to the limitations of any facility with respect to any particular source should be fully countered before any new large-scale facility is contemplated.

Accordingly, our conclusions are: 1) that certain limitations associated with such facilities as open-jet and closed wind tunnels must be overcome before they can provide even the rudiments of a simulation; 2) that there are some types of field, such as discrete tone fan noise and pure jet mixing noise, which should permit simulation at small scale on the basis of present knowledge, and that there is an urgent need for the relevant experiments and accompanying theory to be carried out; 3) that there are several mechanisms, giving rise to "excess" or "internal" noise, for which no scaling laws or prediction schemes are available and for which no simulation will be feasible until more fundamental work on scaling and modeling has been conducted; and 4) that simulation appears to be permanently out of the question for all effects (such as molecular diffusion and relaxation, turbulent scattering, and nonlinear propagation) which appear to be inextricably coupled in the atmospheric propagation process.

II. Theoretical Prediction of Flight Effects in Simple Cases

The aim of this section is to summarize the known theoretical results concerning the effect of flight on various types of acoustic source, and thereby to show how varied and surprising many of the results are. Most of the results are in fact the outcome of very recent work.

In order not to blunt the main thrust of the argument, the details are relegated to the Appendix. The message to be drawn from this discussion is that flight effects on the noise field of a source depend sensitively on the detailed nature of the source and its acoustic environment, as well as on the direct result of flight on the source strength. Prediction of flight effects is possible only for a few especially simple cases,

and for those the effects are surprisingly large. For most sources of interest in aeroacoustics there is no possibility of predicting flight effects at present, though the directions in which research should proceed are fairly clear. Any flight simulation facility must recognize the importance of the detailed nature of the noise source and its acoustic environment and must preserve these features in an appropriately scaled form. Inasmuch as these features differ considerably between the various types of source and environment, it is quite clear that effective flight simulation will be dependent on the faithful modeling of the sources involved.

III. Flight Simulation Facilities

This section gives a brief description of some advantages and disadvantages of various frequently used modes of flight simulation. The discussion does not consider any particular kind of noise source, and in many cases the disadvantages listed involve difficulties which are not specific to the problem of flight simulation. In such cases the possible errors and corrections which may have to be applied are not quantified further. We confine attention in detail to issues raised by the particular mode of flight simulation, and we warn the reader against too literal a correlation of the advantages or disadvantages of a facility with the space taken here to describe them.

Tracked Vehicles

Advantages

In principle, this method offers a proper simulation free from all corrections associated with a limited volume of mean flow around the source. For full-scale aero-engines and large-scale assemblies it is likely to be the only satisfactory alternative to actual flight. Potential problems associated with monitoring the position and speed of the vehicle and correlating the position with the sound field are very much easier to solve with a tracked vehicle than in the real flight case.

Disadvantages

The method suffers from the difficulties which obtain in the real flight case, associated with a transient rather than steady-state situation. There are further problems concerning the self-noise of the vehicle, the limited range of constant vehicle speed and, most importantly, problems of ground reflexion that here are more acute than in any other simulation. Whether ground reflexion effects can be satisfactorily eliminated by the use of combinations of microphones mounted on very high supports and microphones mounted flush in the ground remains to be seen.

Spinning Rig

Advantages

The primary advantage of a rotating arm simulation of this kind lies in its versatility as a research tool. The use of concrete aprons and flush-mounted microphones should enable ground reflexion effects to be completely eliminated at spinning rig scales.

Disadvantages

There are unresolved difficulties in the Rolls-Royce spinning rig connected with the repeated passage of the jet through its own wake and with calibration of the nozzle pressures in the model jet. The transient nature of the field is also troublesome, as is the worry that path curvature might have significant but unforeseen side-effects. The attachment of the model to a lifting arm and the consequent interaction of the jet with an asymmetric secondary flow also introduces further differences from the full-scale situation.

Open-Jet Wind Tunnel (Measurement in the Static Fluid beyond the Jet Shear Layer)

Advantages

There are no difficulties associated with measurement of unsteady pressure fields and no difficulties, in principle, in getting to the very distant far-field. There are potential ground reflexion problems but these can usually be eliminated by suitable anechoic treatment.

Disadvantages

Aerodynamic corrections are needed to account for the free shear layer. Corrections for transmission across such shear layers may be large – indeed work by Jacques² indicates corrections of 5 dB or more in important parts of the field and even then are subject to the following uncertainties. 1) Finite thickness of the shear layer is neglected. 2) Axial spreading of the shear layer is neglected. 3) There is uncertainty as to whether single shear-layer transmission represents an adequate correction. 4) Scattering from shearlayer turbulence is neglected and the sound assumed to be merely refracted through a laminar model of the mean flow. 5) Instability issues and the possible generation of sound by open-jet shear layers when forced by the sound of the primary source are completely unresolved. 6) If the primary sources are extensively distributed and the correction functions are rapidly varying with angle, it will be necessary to go to the very distant far-field where the whole jet flow, as viewed through the tunnel shear layer, can be regarded effectively as a point source. Take, for example, the plane shear layer, for which the wind-tunnel/flight-transmission correction is given by Jacques. 2 Even for small values of the flight Mach number M_a the correction functions vary rapidly near the zone of silence in the forward arc. Suppose that jet mixing noise measurements are taken at a flight Mach number of 0.3, at an angle $\theta = 120$ deg with respect to an origin at the nozzle exit plane. Then, if the correction function is not to vary by as much as 3 dB over the effective length (20 D say) occupied by jet noise sources, the observation angle $\tilde{\theta}$ must be less than 135 deg with respect to an origin 20 D downstream from the nozzle. This sets a minimum measurement distance of 60 D from the tunnel shear layer in order for the transmission correction (which itself is of the order of 4-7 dB here) to apply uniformly to within 3 dB to all the jet noise sources, while uniformity to within 1 dB increases this distance to over 120 D. 7) Jet noise from the free shear layer needs to be estimated – possibly by running the tunnel with no primary excitation, though this will only be unambiguous if unstable response to the primary excitation can be excluded. Unstable response of the tunnel flow is only to be expected, on the basis of present knowledge, if the forcing has significant energy organized in axisymmetric modes with Strouhal numbers (based on tunnel speed and diameter) between about 0.2 and 0.8. In the case of a 3-in. jet in the 24-ft RAE tunnel this possibility can be entirely discounted. 8) The size of the openjet flow is subject to the constraints that it should be large enough so that no primary source near-field can interact with the open-jet shear layer and that the dynamics of the primary source should take place within a uniform open-jet flow, and that it should be small enough for measurements to be taken in the far-field of the open jet as a whole.

Jacques ² has considered the transmission corrections in great detail for the case of a harmonic point source (radiating a wavelength λ at rest). The model he uses is based on earlier work by Howe, in which, if the vortex sheet is plane, the source lies a distance h from it, while if the vortex sheet is of circular section, it has radius h and the source lies on the axis. For these situations Jacques calculates the transmission characteristics and displays them as wind-tunnel/flight corrections for various values of h/λ , flight Mach numbers M_a , and angles θ measured from the flight direction. It is found that the corrections certainly exceed 3 dB over much of the field at values of M_a and h/λ of interest. This, and the fact

that the corrections are subject to a number of uncertainties, would appear to exclude the open-jet tunnel at present as a reliable simulation facility. Against this it should be recognized that the SNECMA and ONERA workers are well aware of the possible inaccuracies and have suggested the open-jet tunnel as potentially the best simulation; that Jacques³ has in fact obtained good agreement between JT-12 engine flight noise levels as measured in a flight simulation involving the Bertin Aerotrain and levels of a 2-in. model jet sheathed by a 6-in. coaxial jet with corrections appropriate to a single plane vortex sheet; and that RAE work on vortex refraction with the HP115 aircraft also made successful use of the 24-ft tunnel in the open-jet mode.

Before such work can rest on a really secure foundation there is a need for estimates of the sound scattering from the shear-layer turbulence, for experimental study of the instability of a single plane shear layer (the high-frequency counterpart of the well-known axisymmetric instability of Crow and Champagne⁴), for consideration of sound transmission through shear layers of finite thickness and axially changing thickness, and for comparison of the transmission coefficients for a hypothetical point source with those for more realistic sources of the kind described in the Appendix. There is also a need for further comparison of results for plane and circular vortex sheets to establish the sensitivity of the corrections to the form of the model.

Closed Wind Tunnel (Measurements in the Tunnel Mainstream)

Advantages

Inasmuch as the tunnel is properly anechoic, there are no ground reflexion difficulties and not usually any associated with shear-layer transmission. (In the case where measurements are made in the mainstream of an open-jet wind tunnel there *are* the questions of reflexion from the open-jet shear layer, scattering by the shear-layer turbulence, etc. It is probable that the *reflexion* problem is not serious, for the reflexion coefficient of a plane vortex sheet for the speed range of interest is always very small.) Representation of the correct aerodynamic conditions is simpler and more accurate with a closed tunnel than with an open-jet tunnel.

Disadvantages

The anechoicity of the tunnel may be doubtful. Often there is "howling" from the surface lining of the tunnel which, though of high frequency in full-scale terms, may overlap with jet noise at model scale. It is probable that the anechoic wall lining destabilizes the tunnel flow, but it is apparently possible to overcome this. If part of the tunnel wall is formed by an open-jet shear layer, there is again the issue of the selfnoise of this shear layer, and of whether that self-noise is independent of the primary noise field under study.

There are considerable difficulties of principle associated with the calibration of unsteady pressure sensors in a moving stream, though workers at the AVA Göttingen claim to have resolved these difficulties even at tunnel-flow speeds as high as 200 fps. We feel, however, that there are fundamental issues here which have yet to be adequately resolved. The corrections due to mean flow on the response of a microphone calibrated in static fluid are not known, yet seem likely to be of just the same order, and indeed type, as those of interest in the in-flight noise problem.

A very serious limitation on the use of in-tunnel measurements for jet noise study is set by the need to get deep into the far-field. Distances of 50-80 nozzle diameters from the exit plane have been widely used in the past as a compromise between the need to get into the far-field with inverse square law spreading without getting so far from the source that corrections for atmospheric attenuation become excessive. Recent work by Kinns⁵ has, however, shown that the levels at ranges of 50 D and 70 D can differ by far more than the prediction of the inverse square law, as might well be expected in circumstances where the jet noise sources may be

Table 1 Possible noise sources in flight

Source	Positively identified?		Given an accurate full-scale static measurement, can flight levels be predicted	Has it a known counter- part at smaller length
	Statically	In flight	to within 3 dB?	scales?
.1 Compressor/fan discrete			_	
tones	Yes	Yes	Probably ^a	Yes
.2 Compressor/fan broadband	Yes	Yes? b	Possibly ^c	Yes?
			No? ^d	No?
3 Buzz-saw	Yes	Yes	No	Yes? c
4 Inlet (howl, etc.)	Yes	Yes	No	No
5 Engine casing vibration	Yes?g	No	No	No
6 Waves generated upstream of				
nozzle exit	Yes	Yes	Probably ^f	No
7 Waves generated in vicinity			-	
of nozzle exit	Yes?g	No? h	No	No
8 Subsonic jet mixing noise,				
conical nozzle	Yes	No	Possibly i	Yes
9 High-speed jet mixing noise	Yes	Yes j	No	Yes
10 Jets from unusually shaped				
suppressor nozzles	Yes	Yes ^k	No!	Yes
11 Shock-cell noise	Yes	Yes	Yes	Yes
12 Airframe self-noise		Yes		Yes ^m
13 Jet/airframe interaction	Yes	, Yes	No	Yes ^m

^a Flight corrections appropriate to convected dipole probably adequate. ^b Hard to distinguish from spectrally broadened tones. ^c Corrections as in (a) for self-generated turbulence. ^d Corrections as in (a) may not be adequate for inlet and ingested turbulence. ^e Buzz-saw may be scaled in principle, but not in practice because phenomenon depends on fine-scale geometrical details. ^f See Ref. 23. ^g Experiments on Olympus 593 engine. ^h Possibly detectable in spinning rig experiments with and without flow straightener in jet pipe. ^f Given one flight measurement, e.g. the 90-deg spectrum. ^f Certainly on rockets. ^k Certainly in high-speed case. ^f Convected dipole correction adequate, see Ref. 7. ^m If aerodynamically correctly scaled.

Table 2 Possible propagation mechanisms associated with flight

Mechanism	Positively identified?		Given an accurate full- scale static measurement can flight levels be predicted	Has it a known counterpart at smaller length
	Statically	In flight	to within 3 dB?	scales?
2.1 Wing shielding 2.2 Secondary flow	Yes	Yesa	Yes? b	Yes
(vortex refraction, etc) Atmospheric propagation, in- luding:		Yes ^c	No	Yes?
3 Molecular diffusion and relaxation effects	Yes	Yes	Yes	No ^d
.4 Turbulent scattering	Yes	Yes? e	No	No ^d
.5 Nonlinear propagation	Yesf	Yesf	No	No ^d
6.6 Ground absorption	Yes	Yes	Yes	No ^g

^a On HP115 and VFW614 aircraft. ^b Correction given in Appendix probably adequate if sources are unchanged by flight. ^c Experiments with VC-10 aircraft. ^d These propagation mechanisms seem inseparably related at present, and each follows different scaling laws. ^c Crackle on Olympus 593 due to turbulent focusing, see Refs. 8 and 9. ^l Possible interpretation of recent Concorde data. ^g Scaling of ground roughness is unknown.

distributed over an axial distance of 30 D. This is confirmed by results from SNECMA. SNECMA had been able to reconcile the discrepancies to a considerable extent by using model data to infer an effective axial location of the source of any particular frequency, and then predicting the levels of 70 D from those at 50 D on the basis of inverse square law spreading of each frequency from its effective source. It seems possible that a range of some 200 D from the nozzle is necessary for the jet mixing noise sources to appear to be effectively collected together at one point. This constraint, which is necessary if tunnel measurements are to be scaled unambiguously, will evidently be very hard to meet in a wind-tunnel facility.

This prompts the question of whether or not it might be possible to use near-field pressure measurements with an extrapolation to the far-field in order not to have to face the uncertainties posed by lack of range within the tunnel. We suggest that techniques which have successfully been used in

underwater acoustics (where range problems set far more severe constraints) might well be developed for aeronautical applications as a much less costly and potentially less ambiguous alternative to increasing the diameter of an acoustic wind tunnel. The methods rely on the fact that the far-field sound pressure at frequency ω and at any angle is determined by the component of the wavenumber-frequency Fourier transform of the near-field pressure at the frequency ω and at the wavenumber corresponding to propagation of a plane sound wave in the observation direction,

$$p_{\text{far-field}}(\mathbf{x},\omega) \propto \tilde{p}_{\text{near-field}}(\mathbf{k} = -\frac{\omega}{a_{\theta}}\hat{\mathbf{x}},\omega)$$

 \hat{x} being a unit vector in direction x, a_{θ} the sound speed, and the tilde denoting a spatial Fourier transform. The near-field measurements may be made at any position outside the mean jet flow such that the homogeneous wave equation holds everywhere from this position out to the far-field observer at x. Maidanik and others have developed methods of

[§]Private communication from R. G. Hoch, 1975.

measuring $\tilde{p}(k,\omega)$ directly through microphone arrays phased as a spatial or wavenumber filter. The possibilities of using such arrays in aeronautical work should be investigated, starting with experiments to compare the predicted and directly measured far-field levels in a well-documented situation (on a clean model jet rig for example). This kind of method has the further advantage that it does not involve any correction of measured far-field data to account for atmospheric attenuation or possible nonlinear propagation effects which do not necessarily change in any known way under change of scale.

In terms of the issues discussed in this section, we feel that the closed wind tunnel offers, potentially, the most reliable simulation because the issues which must be resolved for the closed wind tunnel are quite clear. The tunnel must be made adequately anechoic; the problems of measuring pressure and calibrating pressure sensors in the presence of mean flow must be overcome; and a "wavenumber filter" directional microphone array must be developed so that the difficulty of getting into the very distant field can be sidestepped in favor of the use of near-field pressure measurement.

There seem to be many more issues to be clarified before open-jet tunnels can provide a reliable simulation. Many of these issues, however, will set less of a problem when the scales of the tunnel flow and the model jet flow are well separated, as is the case in the present experiments using a 3-in. jet in the 24-ft RAE tunnel.

IV. Noise Sources and Propagation Mechanisms in Fight

The discussion has not yet turned to the issue of whether the different facilities allow a proper simulation of sound sources and their acoustic environment, yet it was noted in Sec. II that it appears to be important for the simulation to model these faithfully if flight effects are to be accurately reproduced. Accordingly, it is now necessary to consider what sources are likely to be involved in the full aeroengine problem and what is known about them with regard to flight effects and scaling to smaller dimensions. Tables 1 and 2 attempt to set out this information; they are not intended to be precise but to highlight areas of ignorance, especially in the static engine noise problem, and to show the issues which must be resolved if a fully satisfactory flight simulation is ever to be achieved. The entries in the tables are qualified briefly wherever the source of information is not well known, and wherever they may be regarded as a personal opinion.

V. How Can Flight Effects be Simulated?

We try now to answer this question, assuming that the difficulties raised in Sec. III have been overcome, and concentrating on the modeling and scaling problems which are posed for each of the sources and processes of Tables 1 and 2 by any attempt at flight simulation. The numbers listed in parentheses for each subsection refer to the noise sources listed in Tables 1 and 2.

Compressor and Fan Discrete Tones (1.1)

If the tip Mach number is constant, the Helmholtz number $kL = (2\pi f L/a_0)$ is also constant (assuming that Reynolds number effects are unimportant), and then the acoustics of the tone generation process are preserved in any small-scale modeling with the same geometry. The dynamics are likely to be only a weak function of Reynolds number, and therefore the process can be properly modeled at small scale. In that case flight simulation can be achieved with any facility.

Compressor and Fan Broadband Noise (1.2)

No reliable scaling laws are known, so that no faithful small-scale simulation is possible. The only alternative to actual flight involves a full-scale simulation facility such as a tracked vehicle or large wind tunnel. If the self-generated turbulence partly responsible for the broadband noise is not

significantly changed by flight, the flight levels for that part can probably be predicted to within 3 dB given an accurate static measurement and use of the results of the Appendix. For the part of the broadband noise due to inlet turbulence, no prediction of flight levels can be made, and only a fullscale facility can be used for flight tests. These flight levels seem likely to be lower than those associated with selfgenerated turbulence, and therefore the static levels due to self-generated turbulence need to be determined independently of those due to intake distortion. This position may, in fact, be unduly pessimistic. If Reynolds number effects are small, and if the self-generated turbulence arises from natural transition, geometric scaling should work. Inlet turbulence could also be produced at suitable scales and intensities to permit small-scale simulation. There are, however, unknown factors, such as the influence of annulus boundary layer and blade-tip clearance gaps which make full-scale work preferable at present.

Buzz-Saw, Inlet Howl, and Casing Vibration (1.3-1.5)

In all of these cases, as in 1.2, there are no scaling laws, so there is no alternative to flight simulation at full scale by means of tracked vehicles, etc.

Waves Generated Upstream of, or in the Vicinity of, the Nozzle Exit (1.6, 1.7)

Here there are difficulties of modeling which apply to all situations except that involving the complete full-scale aeroengine. The difficulties center on the fact that, although it may be possbile to generate the actual nozzle exit plane fluctuations which occur in the complete engine with the aid of suitable tail-pipe sources in a model, it is not clear that these statically equivalent sources will suffer the same flight effect as the real engine sources. Moreover, it is not clear that, even under static conditions, the appropriate exit-plane fluctuations can be generated by equivalent sources when the jet flow is present in the tail pipe and exhaust - for that presupposes that the jet noise field remains unaffected by possible response of the jet to tail-pipe forcing. For example, if exit-plane forcing were accomplished by a loudspeaker to simulate internal engine noise at Strouhal numbers around 0.5, the unstable jet response at such frequencies would lead to very large variations in the distant jet noise fields for very small variations in the level of tail-pipe noise. This may not be an objection at the rather high frequencies (2-8 KHz say) typical of full-scale internal engine noise generated by sources deep within the engine (source 1.6), for there is no evidence of jet instability at these frequencies, but it is a real possibility at lower frequencies where the source may either be deep within the engine or within a wavelength of the exit plane (source 1.7).

At present, therefore, there is no reliable simulation of "internal noise" or "excess noise" even under static full-scale conditions. Neither is there any possibility of predicting the flight levels to within 3 dB when given an accurate full-scale static measurement (though see the discussion of Sec. A.3 of the Appendix and Ref. 23). The only experiments for which a reasonable interpretation of the results could at present be given are ones involving a full-scale static/flight comparison involving precisely the same "excess noise" sources—for example, static and flight tests on the same engine under "excess noise"-dominated conditions.

Flight effects on "excess" and "internal" noise fields are currently a matter of great concern, and it is regrettable that the flight simulation possibilities seem slim at present. We do not think that the objections raised here are academic; the difficulties in the way of flight simulation of excess noise at model scale are just a disguised version of the reasons why excess noise remains an intractable problem in the static engine case. It may well be, however, that scaling laws for internal and excess noise can be found, and that it will be possible to realistically simulate excess noise sources under

flight conditions. When that has been achieved a study of flight effects at model scale will be as worthwhile and feasible as it now is for compressor tones or pure jet mixing noise, say.

Pure Jet Mixing Noise (1.8, 1.9)

Here the scaling laws are known even in the case of heated jets, and any method of simulation can, in principle, be used (though the arguments of scale in Sec. III may militate against the closed wind tunnel, in that the sources may be distributed over 30 diam of exhaust flow, and a minimum distance of 200 D may be necessary to meet the far-field requirement). No reliable experiments appear to have been carried out yet on the effect of flight on pure mixing noise from clean model jets, neither has an adequate theoretical study yet been published. Nonetheless, an extension of Mani's work 10 (a development of the Lighthill theory in which flow-acoustic interaction is accounted for explicitly with the aid of a simple slug flow model, allowing for differences in mean density and sound speed between the jet flow and ambient) should probably give an estimate of the pressure spectrum at any angle, frequency, jet, and flight condition accurate to within 3 dB or so. Such a theoretical prediction will need the provision of one measure of the "source strength" under flight conditions at one particular jet and flight condition, and from this the spectrum can probably be determined at all other angles and conditions. There is an urgent need for relevant theory and experiment here to be carried out, for until this is done the "floor" above which "excess" noise dominates the flight noise is unknown, and the interpretation of experimental data becomes entirely a matter of conjecture.

Jets from Unusually Shaped Suppressor Nozzles (1.10)

For fixed geometry the noise fields from clean jets of this kind scale in the same way as those from convergent nozzles. There are no difficulties in the flight simulation at smaller scales, but the effects of flight are known to be surprising, and there is at present no possibility of predicting the levels from static data (though, if the extension of Mani's theory to the flight case proves successful for jets from round convergent nozzles, there is some hope that the flight levels for suppressor nozzles might be predicted on the basis that the nozzle effect was predominantly to change the fluid shrouding of noise sources which are essentially unchanged from the case of a convergent nozzle).

Shock-Cell Noise (1.11)

Scaling laws for broadband shock-cell noise have been given by Harper-Bourne and Fisher¹¹ on the basis of model data. Extrapolation to Olympus 593 scales of the model data gave good agreement for the limited range of 593 conditions at which shock-cell noise was clearly identifiable. The degree to which shock-cell noise is affected by changes in internal engine flow is not known, however, so that it is not certain that an adequate simulation of full-scale engine shock-cell noise can be achieved on a clean small-scale rig. Assuming that that is possible, any method and any scale may be used for flight simulation. Flight effects cannot yet be predicted. The shock-turbulence interaction sources are fixed relative to the aircraft, and so would be expected to suffer some form of Doppler amplification ahead of the aircraft; and in fact this always seems to occur in flight tests of imperfectly expanded supercritical jets. The details of the amplification may be expected to be complicated, however, in view of the results of the Appendix and will depend on a much more detailed analysis of the shock-turbulence interaction in this particular context than has yet been published. The issue is further complicated by lack of knowledge of flight effects on the cellular jet structure and on the eddies which collide with the shocks to generate the noise. Nonetheless, it appears from recent SNECMA work⁷ that a convected dipole flight correction factor is probably adequate for most applications.

Airframe Self-Noise and Jet/Airframe Interaction (1.12, 1.13)

These sources may be simulated at smaller scales provided that the aerodynamic interactions are correctly simulated. which will usually amount to a Reynolds number criterion which may be difficult to meet at small scales. If it can be met, the source mechanism is properly preserved in the scaling, and flight effects can be simulated with any facility. Prediction of flight effects is not yet possible. The acoustic effects are likely to be quite large and complicated, if the simple models described in the Appendix are any guide, though if any of these models is relevant to the problem of jet-airframe interaction, then the results quoted there will probably be adequate for prediction to within 3 dB. One can, for example, certainly argue that the interaction of the near field of a quadrupole source with a semi-infinite flat plate is indeed a model for the interaction of a jet flow with a wing or flap whose chord is large compared with the wavelength. If, further, one can argue that flight effects on the quadrupole strength are small, then, as described in the Appendix, flight effects on the interaction field should be represented simply by a factor $(1 + M_a \cos \theta)^{-5}$ on the sound pressure level (SPL), M_a being the aircraft Mach number and θ the angle from the downstream direction. If the chord is not large compared with the wavelength, a much more complicated situation is likely to arise in which, again, an increase in SPL in the sideline direction $\theta = 90$ deg cannot be ruled out.

Wing Shielding (2.1)

Simulation of wing shielding of overwing engine sources is possible at small scales subject only to preservation of the ratio of dominant wavelength to boundary-layer thickness, etc. (if it is thought that boundary layer or wake refraction is a significant part of the shielding). If the noise sources can be represented by an equivalent set of distant point multipoles, the flight effect on the shielding mechanism can be calculated (Appendix), though Doppler factors appear in this case to a smaller power than in any other. Flight effects on wing shielding can therefore be simulated in any appropriate facility. Very crude estimates indicate that the flight effects for this mechanism are small, and probably represented by no more than a factor $(1 + M_a \cos \theta)^{-3}$ on the static SPL.

Secondary Flow, Vortex Refraction, etc. (2.2)

The essential requirement here is that the simulation at small scale should preserve dynamical similarity (or some appropriate Reynolds number) and also the Helmholtz number (vortex scale)/(wavelength). Otherwise there are no obstacles to small-scale simulation (assuming that these scalings do not conflict with those required for whatever primary source field is used – for example, a jet exhaust).

Molecular Diffusion and Relaxation, Scattering by Atmospheric Turbulence, and Nonlinear Propagation Effects (2.3-2.5)

The role of these effects, even in full-scale static tests, is not clear. Turbulence scattering and nonlinear propagation have invariably been neglected in the past, while corrections for diffusion and relaxation effects have always been applied on the basis of experimentally established attenuation curves appropriate to various meteorological conditions. In the past four years the theory of wave scattering by turbulence has advanced to a state in which a realistic assessment of scattering effects due to atmospheric turbulence can probably be made, and this needs to be done, if only to definitely exclude turbulence from the propagation issue, which is now becoming increasingly clouded. Recent Concorde studies (and, in fact, also much older BAC studies on 1-11 and VC10 noise levels) indicate that the usually accepted corrections for atmospheric absorption can display alarming deficiencies above 1 KHz or so, and lead to a large overprediction of the spectral level at 10 KHz, relative to any "sensible" anticipated spectral level. Accompanying this feature is a fairly clear indication of significant nonlinear propagation effects, which

have previously always been discounted because of the relatively low SPLs in jet noise (145 dB within a few feet of the Olympus 593 for example) and because of the spherical spreading which is expected to largely offset nonlinear wave steepening. The whole issue of anomalous atmospheric attenuation is now the subject of intensive study, and it is inappropriate to go further into the matter here. For our purposes it is enough to note that there is no scaling at all in regard to real and possibly nonlinear atmospheric propagation effects, as matters now stand at any rate, and these effects can only be effectively studied experimentally in the full-scale real flight situation.

Ground Reflexion (2.6)

This again is an effect which can only be studied at full scale unless all measurements are reckoned relative to perfectly smooth ground. Otherwise the necessary scaling of ground roughness relative to sound wavelength, and hence of ground impeda :ca as a function of frequency, cannot be made.

VI. Conclusions and Recommendations

The primary aim of the present report is to make the following points quite clear.

The real difficulties in the way of successful flight simulation of aeroengine noise run much deeper than the issues (such as wind-tunnel anechoicity) which are obvious and to which a great deal of attention is already being paid. The real obstacles are fundamental ones set by inadequate knowledge of the source and propagation mechanisms, under both static and flight conditions, and of inadequate knowledge of the behavior of the sources and their noise fields under change of scale. Progress in flight simulation requires work in the following three directions: 1) development of simulation facilities at different scales in which at least all the objections raised in Sec. III have been satisfactorily met; 2) study, in simulation facilities, at the earliest opportunity of flight effects on those noise sources which allow simulation at smaller scales, and correlation of the experimental measurements with theoretical estimates; and 3) fundamental work on the scaling and simulation of "excess" and "internal" jet noise sources, broadband fan noise sources, etc.

Specifically, the experiments which are currently feasible, granted the existence of a suitable wind-tunnel facility, for example, free of the limitations described in Sec. III, involve subsonic jet mixing noise from convergent or suppressor nozzles on a clean model rig, discrete compressor tones, shock-cell noise, and wing shielding. For each of these there is either a relevant theoretical estimate of the flight effect already available, or there are possible extensions of existing theory which should be capable of providing such an estimate. For example, the text describes a possible extension of Mani's fluid-shrouding theory ¹⁰ which should be carried out to give a prediction of flight effects on subsonic mixing noise, and there are corresponding calculations which should be performed for each of the sources mentioned earlier.

As far as the simulation and scaling needs are concerned, these involve principally the sources of "excess" and "internal" noise. Simulation of real engine conditions in model rigs is often attempted in a very crude way by the injection of turbulence and noise into the tail pipe, though no attempt has yet been made to conduct a detailed simulation in this way, even with regard to level and typical frequency. Experiments, therefore, need to be carried out to determine the feasibility of simulation of internal tail-pipe disturbances by controllable sources of noise and turbulence, though there is, in view of Sec. II, still the possibility that fictitious sources which adequately reproduce a certain static field may not suffer the same flight effects as the actual sources. The alternative to simulation involves the reproduction of the internal dynamics of the engine at model scale, and then, apart from practical difficulties, there are large uncertainties associated with lack of knowledge of the relevant scaling laws. For example, the strut noise so clearly observed on a cold turbine rig by Bryce and Stevens¹² has not been detected in complete engines, presumably because the flow onto the exhaust struts is not properly preserved or scaled in the turbine rig.

Certain fundamental experimental and theoretical studies also need to be conducted to improve the capabilities of wind tunnels for flight simulation. These concern various aspects of sound reflexion from, and transmission through, shear layers and anechoic tunnel walls, the related problems of jet instability and consequent amplification of internal engine noise, and the possibility of direct inference of far-field SPL from "wavenumber filter" measurements of the near-field pressure.

The present position for flight simulation is very mixed, varying from satisfactory in relation to pure mixing noise, discrete compressor tones, etc. (given a suitable simulation facility), through cases involving, say, engine-casing vibration or internal engine noise, where no scaling laws are known at present, to cases in which atmospheric propagation effects are very significant, and for which it seems unlikely that any satisfactory scaling can ever be found. For these there can never be any reliable alternative to full-scale flight; for all other cases there are fundamental studies which need to be conducted as a matter of urgency. Fundamental research must be integrated into the program to supplement the ad hoc studies, and to provide the basic background knowledge that is now so lacking. Then, and only then, will flight simulation become a much easier and less ambiguous matter.

Appendix: Theoretical Prediction of Flight Effects

Model Sources in Isolation

The modification, caused by convection, to the pressure field radiated by a concentrated multipole source singularity is well known¹³; it is that

$$p = p_{\text{static}} / (I - M_c \cos \theta)^{n+1}$$
 (A1)

where M_c is the convection Mach number, θ the angle, from the direction of motion, of the observer at the instant the sound reaching the observer was emitted from the source. The exponent n+1 is related to the multipole order; n=1 for monopoles and dipoles, n=2 for quadrupoles, n=3 for octupoles, etc.

These results are not identical to those found from solution of boundary value problems in which the source singularity is replaced by a small finite surface with prescribed motion. In fact, the following results have been obtained so far in the literature:

Compact convected pulsating sphere: 14

$$p = p_{\text{static}} / (1 - M_c \cos \theta)^{7/2} \tag{A2}$$

Compact convected oscillating axisymmetric body: 14

$$p = \frac{p_{\text{static}}}{(1 - M_c \cos \theta)^4} - \frac{\bar{m} U_c d^2 V / dt^2}{4\pi a_0^2 r}$$
 (A3)

with \dot{m} the incompressible virtual mass, U_c the convection speed, V(t) the oscillatory velocity.

Near-field scattering by a compact body (sphere – Ref. 15; cylinder – Ref. 16):

$$p = p_{\text{static}} / (I - M_c \cos \theta)^3 \tag{A4}$$

Distant-field scattering by a compact body: 15

$$O(M_c)$$
 corrections, dipole and monopole, which do not all vanish at $\theta = 90$ deg. (A5)

Near-field scattering by a rigid half-plane: 15

$$p = p_{\text{static}} / (I - M_c \cos \theta)^{5/2} \tag{A6}$$

Far-field diffraction or shielding by a rigid half-plane: 15

$$p = p_{\text{static}} / (I - M_c \cos \theta)^{3/2}$$
 (A7)

A reasonable conclusion from these results is that previously published results based on the forced-wave equation are misleading, and seem to underestimate convective effects. The real situation demands a proper specification of the source. Only the first few natural problems involving such a "proper specification" have yet been solved, and the results are in all cases surprising.

The relevance of some of these results to issues of current interest is obvious; the near-field scattering formula, Eq. (A6), describes the acoustic effect of flight on the noise of blown flaps for example, while Eq. (A7) for a distant source gives the flight effect on the field radiated by an over-wing engine into the shadow below the wing.

In most practical cases neither the compact nor the noncompact limits are usually relevant. We therefore draw the conclusion that we really have no firm guidelines yet as to the acoustic effects of flight on practical aeroengine noise sources. Theoretical study of many other problems is needed to complement the results just given; yet no reliance can be placed on the general applicability of any particular result in view of the wide variety of results obtained already. However, in many cases the index of the Doppler factor $(1 - M_c \cos \theta)$ lies between 5 and 8, and for such cases flight effects are substantial, amounting to an increase of SPL of around 7 dB at 45% to the flight (convection) direction at a flight Mach number $M_c = 0.3$. Further, these theoretical developments, in particular the results in Eqs. (A3) and (A5), are now indicating that flight effects are not always expressible in terms of the Doppler factor, and therefore that it may be unwise to base conclusions on an assumption that the sound at 90 deg to the jet axis is unaffected by source motion.

Jet Mixing Noise Sources

In the Lighthill¹⁷ model of mixing noise generation, the turbulent eddies are represented as compact quadrupoles, convected at Mach number M_c but radiating directly into the ambient atmosphere. Within this model there are just two possible effects of forward aircraft flight at Mach number M_a on mixing noise. First, there is a purely acoustic effect associated with the rate of increase of the jet length on a fixed wavelength scale because of flight, and this gives ¹⁸

$$\overline{p^2} = \overline{p^2} \operatorname{static} / (1 + M_a \cos \theta)$$
 (A8)

 θ being measured from the downstream (exhaust) direction. Second, the strength and spectral content of the quadrupole eddies may be changed because of the reduced shear across the mixing region. There is insufficient experimental evidence to make any definite statements about the effect of flight on the typical velocities, longitudinal and lateral scales, and frequencies associated with the main noise-producing eddies. Measurements have, of course, often been made on jet turbulence in the presence of an external flow, but the evidence for or against that situation as equivalent to one with no external stream but at a lower exit velocity or at different length scales is conflicting and depends very much upon whether mean velocity or pressure profiles were measured rather than Reynolds stresses or turbulence intensities.

The variation in possible predictions of flight effects on pure jet-mixing noise is rather wide. According to Lighthill's theory, as modified by Ffowcs Williams, ¹⁸

$$\overline{p^2} \sim |x|^{-2} a_0^{-4} f^4 \overline{T^2} l_x l_r^2 L_x L_r^2 (1 + M_a \cos \theta)^{-1}$$
 (A9)

in which f is a typical frequency; \overline{T}^2 is the mean square value of Lighthill's acoustic stress tensor T_{ij} ; l_x , l_r are the axial and transverse integral scales of T_{ij} ; and L_x , L_r are the characteristic axial and transverse lengths of the noise-producing volume. If the mixing region is assumed to contain most of the noise sources, then, according to well-known results of Squire and Trouncer, 19 L_r is unchanged by flight but L_x is lengthened according to

$$L_x \sim D U_I / (U_I - U_a) \tag{A10}$$

 U_J and U_a being the jet-exit velocity relative to the nozzle and the aircraft flight speed, respectively, and D the nozzle diameter. If the further assumptions are made that l_x , l_r are unchanged while

$$\overline{T^2} \sim \rho_J^2 (U_J - U_a)^4 \tag{A11}$$

and that

$$f \sim (U_J - U_g)/l_x \sim (U_J - U_g)/D$$
 (A12)

then it follows that

$$\overline{p^2} \sim \overline{p^2}_{\text{static}} \left(\frac{U_J - U_a}{U_J}\right)^7 (I + M_a \cos\theta)^{-1}$$
 (A13)

which is a prediction given by Ffowcs Williams. 18 It is equally plausible, however, to assume that

$$l_x \sim L_x \sim D \ U_J / (U_J - U_a) \tag{A14}$$

and

$$f \sim (U_J - U_g)/l_v \sim (U_J - U_g)^2/D U_J$$
 (A15)

which, with other assumptions unchanged, gives

$$\overline{p^2} \sim \overline{p^2}_{\text{static}} \left(\frac{U_J - U_{\bar{a}}}{U_J} \right)^{10} (I + M_a \cos \theta)^{-1}$$
 (A16)

Lower exponents of the relative velocity ratio can clearly also be obtained depending on the assumptions made, though it should be emphasized that none of these assumptions, or those made earlier, have any secure experimental or theoretical foundation whatever.

Flow-Acoustic Interaction

It has recently been found 10,13 that flow-acoustic interaction plays a dominant role in mixing noise generation and propagation. The interaction is highly frequency dependent, amplifying eddy convection effects at low frequencies and "shielding" high-frequency eddies so that their acoustic power does not suffer convective amplification. How these effects are changed by flight is not yet known, though Fisher and Szewczyk 20 have used the idea of highfrequency fluid shielding to give a possible explanation of the failure of many mixing nozzle silencers to suffer any relative velocity reduction at all in flight. They argue that such silencers do not reduce the source strength by improved mixing, but instead shield high-frequency sources and inhibit convective amplification. Thus, most of the silencing potential is realized in static conditions and cannot be expected to achieve any further relative velocity effect in flight. Such an explanation might obviously be used to explain the preferential angular suppression achieved statically with noncircular nozzles, and this explanation is supported theoretically by studies of Balsa 21 into convective amplification and fluid shielding in jets of elliptical section. How the basic sources could be so insulated from the exterior is unexplained, but these models are as effective as some others in bringing together various experimental results.

It is clear that theoretical studies should be of value here in assessing flight effects on flow-acoustic interaction, for the existing work on that interaction in static conditions has shown how simple models can give easily interpretable results which very greatly improve the agreement between the Lighthill noise theory and experiment, and it should be a relatively straightforward matter to incorporate flight effects into those simple models. Flow-acoustic interaction also needs to be studied both in static and flight conditions insofar as it affects the fields radiated by other sources, in particular by sources within the engine tailpipe.

This was first done by Mani²² for a two-dimensional model, and now, in a complete manner by Munt, 23 who treats propagation out of a hard-walled semi-infinite circular duct, with different mean flows inside and outside the duct. Munt obtains the exact solution, taking full account of the causality requirement and the instability issue. His predicted directivities for cold flow and static exterior fluid agree very well with measurements by Pinker at NGTE, though for heated jets there are discrepancies at small angles to the jet which have yet to be resolved. As with some of the results quoted earlier in this Appendix, the calculations indicate that internal noise fields may well change at 90 deg in the presence of external flow. Jacques³ has also looked at flight effects on internal noise on the basis of high-frequency ray theory, and finds an unexpectedly strong increase in the forward arc, even at the low Mach numbers that are relevant to aircraft operations near the ground.

Acknowledgment

The views expressed in this paper are those of the authors, and do not necessarily represent the opinions of any official body or organization in the U. K. or elsewhere.

References

¹ Atencio, A. and Soderman, P.T., "Comparison of Wind-Tunnel and Fly-Over Noise Measurements of the YOV-10A STOL Aircraft," NASA TM-X-62166, 1972.

² Jacques, J.R., "Wind-Tunnel Simulation of the Effects of Flight on Radiated Sound," British ARC Current Paper No. 1351, 1976.

³ Jacques, J.R., "Aircraft Flight Effects on High Frequency Sound Emerging from a Constant Area Jet Pipe Flow," *Journal of Sound and Vibration*, Vol. 45, April 1976, pp. 569-582.

⁴Crow, S.C. and Champagne, F.H., "Orderly Structure in Jet Turbulence," *Journal of Fluid Mechanics*, Vol. 48, Aug. 1971, pp. 547-591.

⁵Kinns, R., "The Influence of Source to Microphone Distance on the Apparent Distribution of Sources Derived from the Cross-Spectrum of Microphone Signals," Cambridge University Noise Research Unit Rept. CNRU/RK/46, 1974.

⁶Maidanik, G., "Flush-Mounted Pressure Transducer Systems as Spatial and Spectral Filters," *Journal of the Acoustical Society of America*, Vol. 42, Nov. 1967, pp. 1017-1024.

⁷Drevet, P., Duponchel, J.P., and Jacques, J.R., "Effect of Flight on the Noise from a Convergent Nozzle as Observed on the Bertin Aerotrain," AIAA Paper 76-557, 1976.

⁸Ffowcs Williams, J.E., Simpson, J., and Virchis, V.J., "Crackle': An Annoying Component of Jet Noise," *Journal of Fluid Mechanics*, Vol. 71, Sept. 1975, pp. 251-271.

⁹Obermeier, F., "Sonic Boom Behavior Near a Caustic," AGARD CP-131, Paper 17, 1973.

¹⁰Mani, R., "The Influence of Jet Flow on Jet Noise, Parts I and II," *Journal of Fluid Mechanics*, Vol. 73, Feb. 1976, pp. 753-778 and 779-793.

¹¹ Harper-Bourne, M. and Fisher, M.J., "The Noise from Shock Waves in Supersonic Jets," AGARD CP-131, Paper 11, 1973.

¹²Bryce, W.D. and Stevens, R.C.K., "An Investigation of the Noise from a Scale Model of an Engine Exhaust System," AIAA Paper 75-459, 1975.

¹³Goldstein, M.E., Aeroacoustics, McGraw Hill, New York, 1976.

¹⁴Dowling, A., "Convective Amplification of Real Simple Sources," *Journal of Fluid Mechanics*, Vol. 74, April 1976, pp. 529-546.

¹⁵Crighton, D.G., "Scattering and Diffraction of Sound by Moving Bodies," *Journal of Fluid Mechanics*, Vol. 72, Nov. 1975, pp. 209-228.

¹⁶Howe, M.S., "The Generation of Sound by Aerodynamic Sources in an Inhomogeneous Steady Flow," *Journal of Fluid Mechanics*, Vol. 67, Feb. 1975, pp. 597-610.

¹⁷Lighthill, M.J., "On Sound Generated Aerodynamically. I – General Theory," *Proceedings of the Royal Society of London*, Vol. A.211, March 1952, pp. 564-587.

¹⁸ Ffowcs Williams, J.E., "The Noise from Turbulence Convected at High Speed," *Philosophical Transactions of the Royal Society of London*, Vol. A.255, April 1963, pp. 469-503.

¹⁹Squire, H.B. and Trouncer, J., "Round Jets in a General Stream," British ARC R&M No. 1974, 1944.

²⁰ Fisher, M.J. and Szewczyk, V.M., "Flow-Acoustic Interaction Effects in Jet Noise," British ARC 35 212, N. 897, Jan. 1974.

²¹ Balsa, T.F., "Fluid Shielding of Low Frequency Convected Sources," *Journal of Fluid Mechanics*, Vol. 70, July 1975, pp. 17-30.

²²Mani, R., "Refraction of Acoustic Duct Waveguide Modes by Exhaust Jets," *Quarterly of Applied Mathematics*, Vol. 30, Jan. 1973, pp. 501-520.

²³ Munt, R.M., "Sound Propagation from a Semi-Infinite Circular Duct Carrying Uniform Flow," *Journal of Fluid Mechanics*, 1977 (in press).